Addressing Uncertainty in the Choice of Covariance Function in Gaussian Process Modeling with Bayesian Model Averaging

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Research Objectives

Substantive problem:

- Selecting covariance function for observed Gaussian process data e.g. spatial[1]

Methodological objective:

- Use Bayesian model averaging to account for uncertainty in correlation structure[2]

Gaussian Processes

Correlation between errors determined by Euclidean distance h between locations $s \in D$.

$$Z(s) = \underbrace{\mathbf{x}(s)'\boldsymbol{\beta}}_{\text{deterministic}} + \underbrace{\sigma^2\rho(h) + \tau^2\mathbb{1}(h=0)}_{\text{stochastic}}$$

Covariance functions evaluated in this study:

$$\text{Exponential: } \rho(h) = \exp\left(-\frac{h}{\phi}\right)$$

$$\text{Gaussian: } \rho(h) = \exp\left[-\left(\frac{h}{\phi}\right)^2\right]$$

$$\text{Spherical: } \rho(h) = \begin{cases} 1 - 1.5\frac{h}{\phi} + 0.5\left(\frac{h}{\phi}\right)^3, \text{if } h < \phi \\ 0, \text{ otherwise} \end{cases}$$

Bayesian Model Averaging

The posterior probability for any model l in the set of candidate models K is:

$$\pi(\mathcal{M}_l|y) = \frac{\pi(\mathcal{M}_l|y)\pi(\mathcal{M}_l)}{\sum_{m=1}^{K}\pi(\mathcal{M}_m|y)\pi(\mathcal{M}_m)}$$

The marginal posterior distribution of a parameter θ across K is:

$$\pi(\theta|y) = \sum_{m=1}^{K}\pi(\theta|y, \mathcal{M}_l)\pi(\mathcal{M}_l|y)$$

Monte Carlo Simulation

10 simulated datasets are generated from a model with **exponential** covariance using each combination of the following parameters with the `geoR` package.

$$X_1 \sim \mathcal{N}(0, 1.25) \qquad \sigma^2 \in \{15, 20, 25\}$$

$$X_2 \sim \text{exponential}(4) \qquad \phi \in \{10, 25, 50\}$$

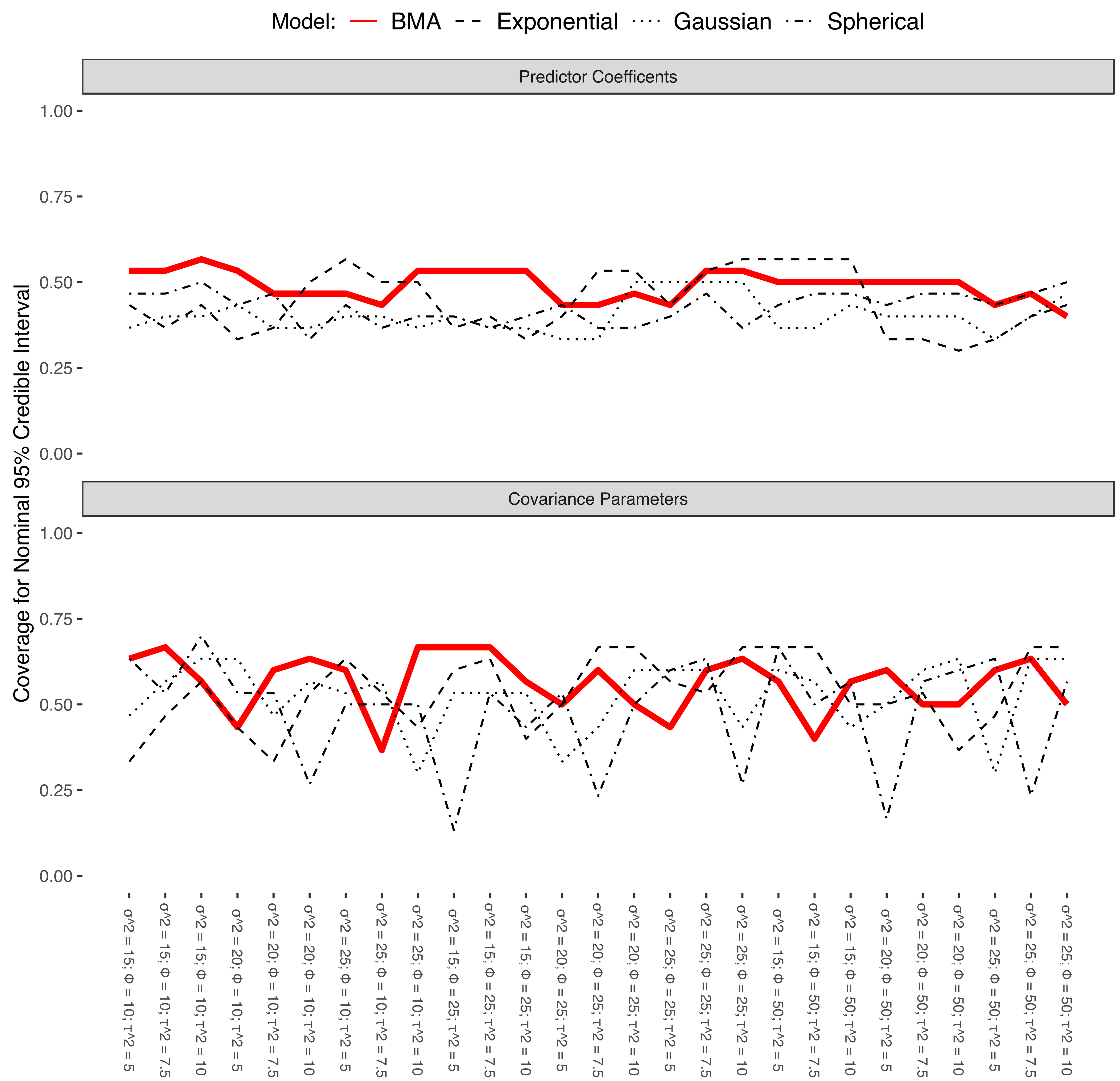
$$\boldsymbol{\beta} = [1.2, 3.5, -2.7] \qquad \tau^2 \in \{5, 7.5, 10\}$$

Model Parameters

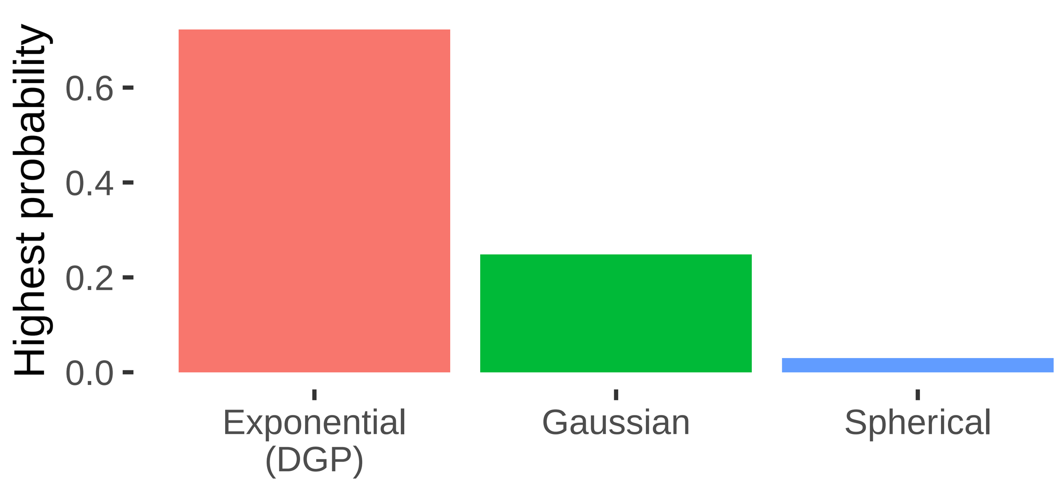
$\mu_\beta \sim \mathcal{N}(0,5)$	$\mu_{\sigma^2} \sim \mathcal{N}(0,5)$	$\mu_\phi \sim \mathcal{N}(0,5)$	$\mu_{\tau^2} \sim \mathcal{N}(0,5)$
$\sigma_\beta \sim \text{Cauchy}(0,2.5)$	$\sigma_{\sigma^2} \sim \text{Cauchy}(0,5)$	$\sigma_\phi \sim \text{Cauchy}(0,5)$	$\sigma_{\tau^2} \sim \text{Cauchy}(0,5)$
$\beta \sim \mathcal{N}(\mu_\beta, \sigma_\beta)$	$\sigma^2 \sim \text{Cauchy}(\mu_{\sigma^2}, \sigma_{\sigma^2})$	$\phi \sim \text{Cauchy}(\mu_\phi, \sigma_\phi)$	$\tau^2 \sim \text{Cauchy}(\mu_{\tau^2}, \sigma_{\tau^2})$

Models are estimated with `Stan` via `rstan`. Marginal likelihoods are estimated using the `bridgesampling` package. Posterior model probabilities are calculated with $\pi(\mathcal{M}_{\text{Exponential}}) = \pi(\mathcal{M}_{\text{Gaussian}}) = \pi(\mathcal{M}_{\text{Spherical}})$ and used to compute averaged point estimates and 95% credible intervals for all parameters in each simulation.

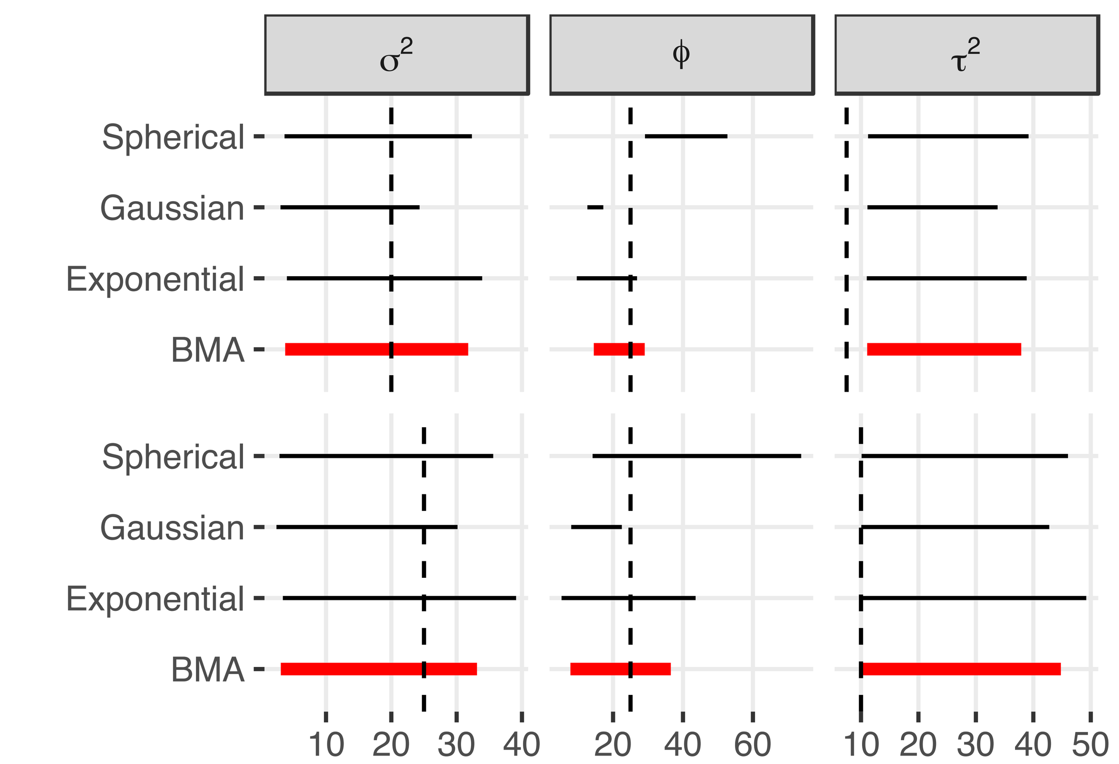
Monte Carlo Simulation Results



Most Likely Model



Representative Simulations



Conclusion

- BMA corrects for inclusion of ill-suited models.
- BMA is less biased, but also less efficient, at estimating covariance function parameters.

Next Steps

- Simulate data from additive and multiplicative combinations of covariance functions at a more fine-grained set of covariance function parameters.

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References

[1] James E. Monogan III and Jeff Gill. Measuring State and District Ideology with Spatial Realignment. *Political Science Research and Methods*, 4(1):97–121, January 2016.

[2] Jacob M. Montgomery and Brendan Nyhan. Bayesian Model Averaging: Theoretical Developments and Practical Applications. *Political Analysis*, 18(2):245–270, 2010.